

**COMPUTATIONALLY EFFICIENT MEANS FOR OPTIMAL CONTROL  
WITH CONTROL CONSTRAINTS**

- [1] This application claims priority to U.S. Provisional Application Serial No. 60/271,792, Filed February 27, 2001.

**BACKGROUND OF THE INVENTION**

**Field of the Invention**

- [2] This invention relates to optimal control of a system. More particularly, this invention relates to active vibration and active sound control systems for the interior of helicopters.

**Background Art**

- [3] Conventional active control systems consist of a number of sensors that measure the ambient variables of interest (*e.g.* sound or vibration), a number of actuators capable of generating an effect on these variables (*e.g.* by producing sound or vibration), and a computer which processes the information received from the sensors and sends commands to the actuators so as to reduce the amplitude of the sensor signals. The control algorithm is the scheme by which the decisions are made as to what commands to the actuators are appropriate.
- [4] A problem may arise in such a control scheme when the control decision yields a command that exceeds the physical capabilities of the system, for example, if the command to an actuator exceeds the actuator's physical limits.

**SUMMARY OF THE INVENTION**

- [5] The present invention constrains commands in a computationally efficient manner while eliminating performance penalties. In the present invention, the control algorithm generates a first command signal. If that first command signal includes a  $k_{th}$  component (for the  $k_{th}$  actuator) that exceeds a maximum allowable value, that component is scaled to or below the maximum allowable value. An anticipated residual vibration is then calculated based upon the change in the  $k_{th}$  component. A second command signal is then generated to try to compensate for the residual vibration (ignoring the  $k_{th}$  actuator, which is already at or near the maximum allowable

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signal). The first command signal (with the scaled  $k_{th}$  component) and the second command signal (with a zero component for the  $k_{th}$  actuator) are summed to yield a third command signal, the components of which may again be compared to the maximum allowable values, and the method performed iteratively before actually sending the command signal to the actuators.

### BRIEF DESCRIPTION OF THE FIGURES

[6] FIG. 1 shows a block diagram of the noise control system of the present invention.

[7] FIG. 2 shows a vehicle in which the present invention may be used.

### DETAILED DESCRIPTION

[8] Active vibration and sound control systems consist of a number of sensors which measure ambient vibration (or sound), actuators capable of generating vibration (or sound) at the sensor locations, and a computer which process information received from the sensors and sends commands to the actuators which generate a vibration (or sound) field to cancel ambient vibration (generated, for example by a disturbing force at the helicopter rotor). The controller algorithm is the scheme by which the decisions are made as to what the appropriate commands to the actuators are.

[9] FIG. 1 shows a block diagram 10 of an active control system. The system comprises a structure 102, the response of which is to be controlled, sensors 128, filter 112, control unit 106 and force generators (also referred to as actuators) 104. A vibration or sound source 103 produces undesired response of the structure 102. In a helicopter, for example, the undesired disturbances are typically due to vibratory aerodynamic loading of rotor blades, gear clash, or other source of vibrational noise. A plurality of sensors 128(a)...(n) (where n is any suitable number) measure the ambient variables of interest (e.g. sound or vibration). The sensors (generally 128) are typically microphones or accelerometers. Sensors 128 generate an electrical signal that corresponds to sensed sound or vibration. The electrical signals are transmitted to filter 112 via an associated interconnector 144(a)...(n) (generally 144). Interconnector

144 is typically wires or wireless transmission means, as known to those skilled in the art.

- [10] Filter 112 receives the sensed vibration signals from sensors 128 and performs filtering on the signals, eliminating information that is not relevant to vibration or sound control. The output from the filter 112 is transmitted to control unit 106 via interconnector 142. The control circuit 106 generates control signals that control force generators 104(a)...(n).
- [11] A plurality of force generators 104(a)...(n) (where n is any suitable number) are used to generate a force capable of affecting the sensed variables (e.g. by producing sound or vibration). Force generators 104(a)...(n) (generally 104) are typically speakers, shakers, or virtually any suitable actuators. Force generators 104 receive commands from the control unit 106 via interconnector 134 and output a force, as shown by lines 132(a)...(n) to compensate for the sensed vibration or sound produced by vibration or sound source 102.
- [12] The control unit 106 is typically a processing module with computing capabilities. Control unit 106 stores control algorithms control memory 105, or other suitable memory location. Memory module 105 is, for example, RAM, ROM, DVD, CD, a hard drive, or other electronic, optical, magnetic, or any other computer readable medium onto which is stored the control algorithms described herein. The control algorithms are the scheme by which the decisions are made as to what commands to the actuators 104 are appropriate. Control circuit 106 is, for example, a microprocessor.
- [13] Measurements of ambient vibration level at a given moment may be assembled in a vector of dimension  $N_{\text{sensors}} \times 1$ , designated  $z$ . The commands to the actuators may likewise be assembled in a vector of dimension  $N_{\text{actuators}} \times 1$ , designate  $u$ . The relationship between a change in actuator commands and the resulting change in sensor measurements may be expressed as the matrix equation  $\Delta z = T \Delta u$ , where  $T$  is a matrix of dimension  $N_{\text{sensors}} \times N_{\text{actuators}}$ . The values of the elements of  $T$  are determined by the physical characteristics of the structure, for example:  $T_{11}$  is the response at sensor #1 due to unit command at actuator #1,  $T_{12}$  is the response at sensor #1 due to a unit command at actuator #2, etc. Many algorithms may be used for

making control decisions based on this model. One such algorithm seeks to minimize the scalar performance index given by:

$$J_{\text{cost}} = \mathbf{z}^T \mathbf{W}_z \mathbf{z} + \mathbf{u}^T \mathbf{W}_u \mathbf{u} + \Delta \mathbf{u}^T \mathbf{W}_{\Delta u} \Delta \mathbf{u}$$

where

$\mathbf{W}_z$  is a diagonal weighting matrix of sensor measurements;

$\mathbf{W}_u$  is a diagonal weighting matrix which constrains control inputs;

$\mathbf{W}_{\Delta u}$  is a diagonal weighting matrix which constrains rate of change of control inputs;

and  $T$  designates the transpose of a vector or matrix.

With this objective, the control decision for the  $i^{\text{th}}$  step takes the form

$$\Delta \mathbf{u}_i = \mathbf{D} [\mathbf{W}_u \mathbf{u}_{i-1} + \mathbf{T}^T \mathbf{W}_z (\mathbf{z}_{i-1})]$$

where

$$\mathbf{D} = -(\mathbf{T}^T \mathbf{W}_z \mathbf{T} + \mathbf{W}_u + \mathbf{W}_{\Delta u})^{-1} \quad \text{Equation (1)}$$

and  $^{-1}$  indicates a matrix inversion.

[14] A problem may arise in such a control scheme when the control decision yields a command that exceeds the physical capabilities of the system, for example, if the command to a force generating actuator exceeds the actuator's physical limits. The strategy for imposing constraints on the commands can dramatically affect control system performance.

[15] One such strategy is to impose command constraints by scaling the commands to actuators by a constant which reduced the maximum command component to the maximum allowable. In other words, if a component of  $\mathbf{u}_i$  is greater than maximum allowable then setting  $\mathbf{u}_i = \mathbf{K} \mathbf{u}_i$  where  $\mathbf{K}$  is selected so that the largest component of  $\mathbf{u}_i$  is equal to the maximum allowable. While this approach was computationally efficient, it often results in a precipitous drop in system performance when constraints are imposed.

[16] Another way in which command constraints may be imposed is by increasing the value of  $W_u$ , which increases the cost of commands in the performance index, and consequently drives down the command amplitudes. The problem with this approach is that when  $W_u$  is selected to keep commands within constraints during demanding operating conditions performance is degraded during less demanding operating conditions by unnecessarily suppressing command amplitude.

[17] Compared to the first method, the performance of the vibration control system is much better for conditions requiring high actuator output, but much worse for conditions requiring lower actuator output, because the increased value of  $W_u$  unnecessarily constrains the actuator commands. An approach to overcoming this problem is to allow  $W_u$  to vary based on command levels, though this introduces additional complexity associated with stability of the control system used to schedule variation of  $W_u$ .

[18] The present invention provides a means for constraining commands in a computationally efficient manner while eliminating performance penalties.

[19] The controller algorithm is modified to do the following:

[20] The change in command is calculated as before in accordance with:

$$\Delta u_i = D (W_u u_{i-1} + T^T W_z(z_{i-1})).$$

Before sending the command to the actuators, the magnitudes of the components of the resulting command  $u_i = \Delta u_i + u_{i-1}$  are calculated and the component with maximum amplitude is identified. If that component for example, the  $k$ th component  $u_{i,k}$  is greater than the maximum allowable, then that component is scaled by a constant to reduce its amplitude to the maximum allowable  $(u_{i,k})_{new} = C u_{i,k}$ , where  $C = |(u_i)_k| / U_{max}$ , and the change in the  $k$ th component in the command is calculated  $\Delta u_{i,k} = (u_{i,k})_{new} - u_{i-1,k}$ . The response to this component of the command is then calculated by:

$$(z_{i-1})_{new} = (z_{i-1}) + T \Delta u_{i,k}.$$

This quantity,  $(z_{i-1})_{new}$ , is the residual vibration which remains after the constrained control component alone is applied to the system. Now a new controller weighting matrix  $W_{u,new}$  is created which is identical to  $W_u$  except that the element associated with weighting the constrained control component,  $W_{u,new,k,k} = W_{u,k,k} + A$  where  $A$  is a very large number. By modifying  $W_{u,new}$  in this way, the participation of the  $k$ th control component is suppressed in attacking this residual vibration. A new command change is calculated with

$$D_{new} = -(T^T W_z T + W_{u,new} + W_{\Delta u})^{-1} \quad \text{Equation (2)}$$

and

$$\Delta u_{i,new} = D_{new} (W_{u,new} u_{i-1} + T^T W_z (z_{i-1})_{new}).$$

The  $k$ th component of  $\Delta u_{i,new}$  will be 0 because of increased value of  $W_{u,new,k,k}$ . Finally, the command to be sent to the actuators is constructed:

$$\Delta u_{i,to actuators} = \Delta u_{i,new} + \Delta u_{i,k}.$$

The procedure can be applied iteratively in case another component of  $\Delta u_{i,to actuators}$  exceeds maximum allowable command amplitude. Compared to the previous methods, performance is very good at both high and low speed.

- [21] The procedure can be rendered computationally efficient if the matrix inversion in Equation (2) can be eliminated. Noting that matrix which is inverted in Equation (1) differs only in one element from the matrix inverted in Equation (2) (the  $k,k$  element), and the elements of  $D_{new}$  can be determined without matrix inversion using the relationship:

$$D_{new\ 1,m} = D_{1,m} - A D_{1,k} D_{k,m} / (1 + A D_{k,k})$$

where  $W_{u,new\ k,k} = W_{u\ k,k} + A$ .

[22] FIG. 2 shows a perspective view 20 of a vehicle 118 in which the present invention can be used. Vehicle 118, which is typically a helicopter, has rotor blades 119 (a)...(d). Gearbox housing 110 is mounted at an upper portion of vehicle 118. Gearbox mounting feet 140 (a)...(c) (generally 140) provide a mechanism for affixing gearbox housing 110 to vehicle airframe 142. Sensors 128(a) through (d) (generally 128) are used to sense vibration or sound produced by the vehicle, which can be from the rotorblades 119 or the gearbox housing 110. Although only four sensors are shown, there are typically any suitable number of sensors necessary to provide sufficient feedback to the controller (not shown). The sensors 128 may be mounted in the vehicle cabin, on the gearbox mounting feet 140, or to the airframe 142, or to another location on the vehicle 118 that enables vehicle vibrations or acoustic sound to be sensed. Sensors 128 are typically microphones or accelerometers. These sensors generate electrical signals (voltages) that are proportional to the local sound or vibration.

[23] The present invention has been described in detail by way of examples and illustrations for purposes of clarity and understanding, and not to in any way limit the scope of what is claimed. Those skilled in the art will understand that certain changes and modifications may be made without departing from the scope of the invention. Alphanumeric identifiers for steps in the method claims are for ease of reference by dependent claims, and do not indicate a required sequence, unless otherwise indicated.